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Estimation of the stagnant thermal conductivity of saturated porous media

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INTRODUCTION

PRASAD *et al.* [1] recently reported new experimental data for the thermal conductivity of liquid-saturated porous beds of spheres. They compared their data with three correlation formulas proposed by Kunii and Smith [2], Krupiczka [3] and Zehner and Schlünder [4], respectively. Each of these formulas is quite complicated and contains at least two numerical constants which either have been determined empirically or are specific to a particular geometry. The purpose of this note is to demonstrate a simple correlation formula which contains no empirical constants and which gives a useful prediction of the thermal conductivity, given the values of the porosity ε, the fluid conductivity k_f and the solid conductivity k_s. Some other aspects of the results of Prasad *et al.* are also discussed.

The weighted arithmetic, harmonic and geometric means of k_f and k_s, with weighting factors ε and $1-\varepsilon$, are denoted here by k_A, k_H and k_G, respectively, and are given by

$$k_A = \varepsilon k_f + (1-\varepsilon)k_s \tag{1}$$

$$1/k_H = \varepsilon/k_f + (1-\varepsilon)/k_s \tag{2}$$

$$k_G = k_f^{\varepsilon} k_s^{(1-\varepsilon)}. \tag{3}$$

Equation (1) gives the appropriate overall conductivity if the heat conduction in the fluid and solid phases is entirely 'in parallel'. Equation (2) is appropriate if the conduction in the fluid and solid phases is entirely 'in series'. One would expect on physical grounds that k_A and k_H should be upper and lower bounds on the overall conductivity k_m of the medium, since a parallel arrangement should offer the least resistance to heat flow and a series arrangement the greatest resistance. The geometric mean of two quantities is always intermediate in value between the arithmetic and harmonic means of those quantities. This suggests that k_G may well be a good candidate for a correlation formula for a general porous medium.

To test this suggestion, these formulas have been applied to the data given in Table 1 of ref. [1], and the results are given in our Table 1. For comparison, we have listed in the last column the values predicted from the formula of Kunii and Smith [2], namely

$$k_{KS} = k_f\left[\varepsilon + \frac{a_1(1-\varepsilon)}{a_3 + a_2\lambda}\right] \tag{4}$$

where $\lambda = k_f/k_s$, $a_1 = 1$, $a_2 = 2/3$, $a_3 = \varphi_2 + 4.63(\varepsilon - 0.26) \times(\varphi_1 - \varphi_2)$, and the quantities φ_1 and φ_2, which depend on ε, can be obtained from a plot in ref. [2]. On the criterion of smallest root mean square relative error, the Kunii–Smith formula is slightly more successful in predicting the data of Prasad *et al.* [1] than the other correlation formulas mentioned above, namely those of Krupiczka [3] and Zehner and Schlünder [4].

We see from Table 1 that k_G is reasonably competitive with k_{KS} as a predictor of the measured data. Prediction becomes difficult when the values of k_f and k_s are greatly different from each other. The root mean square relative error for the k_G values is 2.8 times that for the k_{KS} values, but to balance this disadvantage formula (3) is clearly much less complicated than equation (4), so if a quick estimate of k_m is required then k_G should serve the purpose. Formula (3) is not specific to beds of spheres (as are the other correlation formulas). It is suggested that formula (3) should be useful in estimating the conductivity of general isotropic porous media.

It is true that equation (3) has its limitations, especially when λ is small. In this situation we see from Tables 1 and 2 of ref. [1] that the three correlations represented in ref. [1] are capable of predicting the thermal conductivity quite accurately, whereas our equation (3) leads to an overprediction for each of the systems glycol/steel (Medium 5 in our Table 1), water/steel (for which Table 3 of ref. [1] lists a measured value of 4.653 whereas equation (3) gives 7.61) and water/

Table 1. Data based on Table 1 of ref. [1]. The values of ε, k_s, λ and k_{KS}, and the measured values of conductivity, are those given in that table. The values for k_f have been computed from the two previous columns. The values of k_A, k_H and k_G have been computed from equations (1), (2) and (3), respectively

	Medium	ε	k_s	$\lambda = k_f/k_s$	k_f	k_A	k_H	k_G	Measured	k_{KS}
1	water/glass	0.396	1.10	0.560	0.616	0.908	0.839	0.874	0.837	0.831
2	water/glass	0.425	1.10	0.562	0.618	0.894	0.824	0.860	0.842	0.810
3	glycol/glass	0.349	1.10	0.235	0.259	0.806	0.515	0.664	0.559	0.656
4	glycol/glass	0.427	1.10	0.235	0.259	0.741	0.460	0.593	0.597	0.555
5	glycol/steel	0.416	37.39	0.007	0.262	21.940	0.623	4.746	2.584	2.167
6	glycol/acrylic	0.402	0.16	1.630	0.261	0.201	0.189	0.195	0.221	0.206
7	water/acrylic	0.427	0.16	3.937	0.630	0.361	0.235	0.287	0.479	0.371

lead (for which Table 4 of ref. [1], where the printed value of λ should be replaced by its reciprocal, lists a measured value of 4.78 whereas equation (3) gives 7.87). Thus when λ is small, Krupiczka's formula (formula (3) of ref. [1]), namely

$$k_{\mathrm{Kr}} = k_f \lambda^{-n} \tag{5}$$

where

$$n = 0.280 - 0.757 \log_{10} \varepsilon + 0.057 \log_{10} \lambda \tag{6}$$

is preferable since it is more accurate and is not much more complicated than our equation (3). In fact equation (5) can be written as

$$k_{\mathrm{Kr}} = k_f^{1-n} k_s^n \tag{7}$$

which is of the same form as equation (3) with the porosity ε replaced by $1-n$ (which is, of course, dependent on λ as well as ε).

When $\lambda > 1$, equations (3) and (5) give similar values, which are underestimates of the measured values. Indeed, Table 1 of ref. [1] shows that all the proposed correlations give underestimates. More dramatically, the measured values are anomalous because they exceed by a considerable amount the corresponding values of k_A (which, as noted above, are expected to be upper bounds on the overall conductivities). A similar comment applies to the last entry in Table 4 of ref. [1], which refers to a water/polypropylene system; the measured value of 0.30 exceeds the value of k_A, namely 0.241.

An explanation for the anomaly is not immediately obvious. Prasad *et al.* [1] state that no efforts were made to estimate the heat transfer through the side wall, but the implication is that this was small. In any case, an inwards heat flux is physically implausible, so an explanation must be sought elsewhere. Georgiadis and Catton [5] have shown that a variation of porosity in a packed bed can lead to an increase in a volume flux by a substantial amount (9% in one example) and there is an analogous thermal effect. It is noteworthy that for Medium 6 (in our Table 1) the bed thickness was about 14 times the particle diameter, and for Medium 7 this thickness was only 7 times the particle diameter, and it is known that the presence of smooth walls introduces variations in bulk porosity inside a layer about four diameters thick. Thus variation in porosity can account for at least some of the excess thermal conductivity. However, the anomaly in the case of Medium 7 is so large that it appears that some other explanation is required. One is led to speculate that the experimenters may not have measured a stagnant thermal conductivity, but rather the effective conductivity in a situation in which there were present convection currents set up locally by lateral variations of temperature along the surface of the poorly conducting solid material. Clearly the matter needs further investigation.

After completing the first draft of this note the author realized that he is by no means the first to propose the use of equation (3). Combarnous and Bories [6] refer to an 'empirical model' of Lichteneker which involves this equation.

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